A Well-Behaved $f(R)$ Gravity Model in Planetary Motions

Robab Hashemi

Department of Physics, University of Guilan, P. O. Box: 41335-1914, Rasht, Iran

Reza Saffari [*]

Department of Physics, University of Guilan, P. O. Box: 41335-1914, Rasht, Iran

Abstract

In this paper we consider asymptotic behavior of a hybrid action of $f(R)$ gravity model which proposed by Saffari & Rahvar (2008), in the Solar system scale, which can explain Pioneer anomalous acceleration. We use the resultant weak field gravitational potential which comes from the hybrid action to test its impacts on the Solar system dynamics, by comparing theoretical precession of perihelion of a test particle, $\dot{\varpi}$, with corrections to the standard Newtonian - Einstenian precessions of perihelia of some planets, which recently estimated by Pitjeva (2005a,b, 2006, 2008). Here we show that the asymptotic behavior of hybrid action, are in more accordance with observation relative to the other modifications such as power law and logarithmic corrections (Iorio, 2008). We also show that an extra additional lensing of the prediction of General Relativity is reproduced. Finally we consider the stability condition of planetary orbits in presence of the hybrid action.

Key words: planetary motion, modified gravity, dark matter

1 Introduction

To accommodate some recently observed phenomena (Riess et al., 1998; Perlmutter, 1999; Cole, 2005; Tegmark, 2006; Spergel, 2003, 2007; Clowe et al., 2006; Anderson et al., 1998, 2002), occurring at very different scales ranging from Solar system to cosmological distances, which at present, have not yet

---

[*] Corresponding author

Email address: rsk@guilan.ac.ir (Reza Saffari).

found fully satisfactorily explanations in terms of conventional physics, gravitation or not, (Bertolami et al., 2006; Bertolami, 2008; Lammerzahl et al., 2008), we considering modifications of the standard laws of gravity at different scales; As well as done in recent years. These anomalous effects are: The recent data coming from the luminosity distance of SuperNovae Ia (SNeIa)(Riess et al., 1998), wide galaxy surveys (Cole, 2005) and the anisotropy of cosmic microwave background radiation (Spergel, 2003, 2007) suggest that about 70 percent of the energy density of the present universe is composed by dark energy, responsible for an accelerating expansion. Further more in small scales, the data coming from galactic rotation curves of spiral galaxies (Clowe et al., 2006) can not be explained on the basis of Newtonian gravity or General Relativity (GR), if one does not introduce invisible dark matter. In Solar system there is an unexplained acceleration $a_p = (8.47 \pm 1.33) \times 10^{-10} ms^{-2}$ approximately directed towards the Sun affecting the Pioneer 10/11 spacecrafts after they passed $20 AU$ threshold (Anderson et al., 1998, 2002). In order to reconcile theoretical models with observations, existence of dark matter is postulated in Solar system (Nieto, 2008). The main problem of dark energy and dark matter is understanding their nature, since they are introduced as ad hoc gravity sources in a well define model of gravity. Therefore we find another possibilities, like various modification of gravity. In this paper we are concerned with $f(R)$ gravity theory, where the gravitational lagrangian depends on a function of the scalar curvature R (Nojiri & Odintsov, 2007; Capozziello & Francaviglia, 2008; Faraoni, 2008). These theories are also referred to as 'extended theories of gravity', since they naturally generalize GR: in fact, when $f(R) = R$ the action reduces to the usual Einstein-Hilbert action, and Einstein's theory is obtained. These theories can be studied in the metric formalism, where action is varied with respect to metric tensor and in Palatini formalism, where action is varied with respect to metric and affine connections, which are supposed to be independent from one to another. In general, two approaches are not equivalent: the solutions of Palatini field equations are a subset on metric field equations (Magnano, 1994). Actually, $f(R)$ theories provide cosmologically viable models, where both the inflation phase and the accelerating expansion are reported (de la Cruz-Dombriz & Dobado, 2006; Nojiri & Odintsov, 2009). Furthermore, they have been used to explain the rotation curves of galaxies without need for dark matter (Frigerio & Salucci, 2007). However, because of the excellent agreement of GR with Solar system and binary pulsar observations, every theory that aims at explaining galaxies dynamics and the accelerating expansion of the Universe, should reproduce GR at the Solar system scale, in a suitable weak field limit. Without going into the details, we want to test the model of $f(R)$ gravity and using perturbative approach, we focus on the impacts which modification of the gravitational field due to nonlinearity of the gravitational lagrangian, have on Solar system dynamics. We compare the theoretical prediction for the precession of the longitude of the pericenter of a test particle with the correction to the standard Newtonian-Einsteinian precession of perihelia of some planets recently esti-

mated by Pitjeva.(Pitjeva, 2005a,b). At first, we derive an explicit expression of the secular, averaged over one orbital revolution perihelion precession, induced by anomalous acceleration of asymptotic behavior of hybrid action on the orbits of Solar system planets. Then we will compare obtained results in small eccentricity approximation, with the latest estimated corrections of the perihelion rates. It is important to note that the corrections to the perihelion rates determined in (Pitjeva, 2006, 2008), are phenomenologically estimated quantities of a global, least-square solution in which only Newtonian and Einstenian dynamics was modeled; no exotic dynamical terms were included in the fit. Thus in our opinion such phenomenological corrections can genuinely be used to get information on a hypothetic, un-modeled force. Also, central to the development of the good modified theory of gravity that can describe the phenomenology usually ascribed to dark matter in the analysis of the amount of deflection of light implied by the theory. Then we obtain deflection of light for the hybrid model and compare it with the prediction of GR. This paper is organized as follows: in section 2, we briefly review the theoretical formalism of $f(R)$ gravity, in section 3, we outline a general approach to the perturbations of the gravitational field of GR, in section 4, we introduce asymptotic behavior of hybrid $f(R)$ model and we compare theoretical predictions with observations and compare our results with some models that tested in Solar system, in section 5, we calculate deflection of light, in section 6 we consider stability of circular orbits and finally, conclusions are in section 7.

2 $f(R)$ gravity

Here we introduce the field equation of $f(R)$ gravity (Sotiriou & Faraoni, 2010), we start from modified Einstein-Hilbert action

$$S = \frac{1}{2k} \int d^4x \sqrt{-g} f(R) + S_m. \tag{1}$$

The gravitational part of the Lagrangian, $f(R)$ is represented by a function of the scalar curvature, R. S_m is the matter part of the action. In metric formalism affine connections, Γ's are supposed to be Levi-Civita connections of g and consequently, the scalar curvature R has to be intended as

$$R \equiv R(g) = g^{\alpha\beta} R_{\alpha\beta}(g). \tag{2}$$

On the contrary in the Palatini formalism metric g and affine connections are supposed to be independent, so that the scalar curvature R has to be intended as

$$R \equiv R(\Gamma) = g^{\alpha\beta} R_{\alpha\beta}(\Gamma), \tag{3}$$

3

where $R_{\alpha\beta}(\Gamma)$ is the Ricci-like tensor of the connections Γ. We follow the metric formalism that in this regime, Eq. (1), is varied with respect to the metric $g_{\mu\nu}$, and obtain the field equation of motion

$$F(R)R_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) - kT_{\mu\nu} = g_{\mu\nu}(\frac{1}{2}f(R) - \nabla_\alpha\nabla^\alpha F(R)), \qquad (4)$$

where $T_{\mu\nu}$ is the standard minimally coupled matter energy- momentum tensor and $F = df/dR$. Contraction of the field equation Eq. (4), with metric tensor leads to scalar equation

$$3\nabla_\alpha\nabla^\alpha F(R) + F(R)R - 2f(R) = \frac{8\pi G}{c^4}T, \qquad (5)$$

where T is the trace of the energy-momentum tensor. Eq. (5) is a differential equation for the scalar curvature R. In order to compare the prediction of $f(R)$ gravity with Solar system data, we have to consider the solutions of the field equation Eq. (4) supplemented by the constraints Eq. (5) in vacuum, since tests are based on the observations of the dynamics of the planets in the gravitational field of the Sun. We notice that if $R = constant$ we obtain the Palatini case: so for a given $f(R)$ function in vacuum, solutions of the field equation of Palatini $f(R)$ gravity are a subset of the field equation of metric $f(R)$ gravity. (Multamaki & Vilja, 2006)

3   The proposal method to test modified $f(R)$ gravity model

In this section we outline the general procedure that we are going to apply to the solution of $f(R)$ gravity, In order to compare the prediction of these gravity models with the existing data. In weak field approximation of metric and spherically symmetrical condition

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2\sin^2\theta d\varphi^2 + r^2d\theta^2, \qquad (6)$$

the gravitational (scalar) potential $\phi(r)$ is read from the $B(r)$ function

$$B(r) = 1 + \frac{2\phi(r)}{c^2}. \qquad (7)$$

We expect a gravitational potential in the form of

$$\phi(r) = \phi^N(r) + \triangle\phi(r), \qquad (8)$$

where $\phi^N(r) = -GM/r$ is the Newtonian potential of a point-like mass M in center, and $\triangle\phi(r) \ll \phi^N(r)$ is a correction vanishing for $f(R) \to R$. Now if we have a correction for Newtonian potential, we can calculate the perturbing acceleration and then we will be able to calculate its effect on planetary motions, within standard perturbative schemes (Roy, 2005). Gauss equation for entirely radial perturbing acceleration A_r is

$$\frac{d\varpi}{dt} = -\frac{\sqrt{1-e^2}}{nae} A_r \cos\varphi. \tag{9}$$

After being evaluated onto the unperturbed Keplerian ellipse, the acceleration A_r must be inserted into Eq. (9); then, the average over one orbital period P must be performed. To this end, we use following equations; where $n = \sqrt{GM/a^3}$, is the Keplerian mean motion, a is the planets semi major axis, e is the eccentricity and φ is the true anomaly. (Roy, 2005)

$$\cos\varphi = \frac{\cos E - e}{1 - e\cos E}, \tag{10}$$

$$r = a(1 - e\cos E), \tag{11}$$

$$dt = \frac{1 - e\cos E}{n} dE. \tag{12}$$

In fact, what we aim at, is evaluating the perturbations induced on longitudes of perihelia by the corrections to the gravitational field due to $f(R)$ gravity. In order to compare them with the latest observation we use table 1 and 2 of Pitjeva (2005a,b), that recently processed almost one century of data of different types for major bodies of the Solar System in the effort of continuously improving the $EPM2004/EPM2006$ planetary ephemerides, (Pitjeva, 2006). Among other things, she also estimated corrections to the secular rates of the longitudes of perihelia of some planets of the Solar system as fit-far parameter of a global solution in which she contrasted, in a least-square way, the observations to their predicted values computed with a complete set of dynamical force models including all the known Newtonian and Eisteinian features of motion. As a consequence, any un-modeled exotic force present in nature is; in principle, entirely accounted for by the obtained apsidal extra precession $\Delta\dot{\varpi}$. See table 1 for inner planets and table 2 for outer planets of Solar system.

Now let us briefly outline how we are going to put $f(R)$ gravity on the test. We follow Iorio's regime for testing modified gravity models, (Iorio, 2008) and take the ratios of observational $\Delta\dot{\varpi}$ for different pairs of planets A and B

$$\Omega_{AB} = \frac{\Delta\dot{\varpi}_A}{\Delta\dot{\varpi}_B}, \tag{13}$$

and compare it with the predicted ratios of modified model of gravity ξ_{AB}. If we have $\Psi_{AB} = |\Omega_{AB} - \xi_{AB}| = 0$ within the errors, the $f(R)$ gravity model examined, can still be considered compatible with data, otherwise it is seriously challenged.

4   Asymptotic behavior of the hybrid action

Saffari & Rahvar (2008), use the inverse solution to extract an appropriate action for modified gravity,

$$f(R) = R + \Lambda + \frac{R + \Lambda}{R/R_0 + 2/\alpha} \ln \frac{R + \Lambda}{R_c}, \tag{14}$$

where $R_0 = 6\alpha^2/d^2$ and R_c is the constant of integration, $\alpha \simeq 10^{-6}$, is a small dimensionless constant and $d \simeq 10kpc$, is length scale in the order of galactic size and Λ is cosmological constant. In Solar system, for the range of $R \gg \Lambda$ and $R/R_0 \gg 2/\alpha$, action reduces to

$$f(R) = R + R_0 \ln \frac{R}{R_c}. \tag{15}$$

On the other hand, in galactic scales, for $\alpha \ll 1$ and $R \simeq R_0 \simeq \Lambda$ generic action can be written as

$$f(R) = (R + \Lambda)[1 + \frac{\alpha}{2} \ln(\frac{R + \Lambda}{R_c})], \tag{16}$$

which can justifies flat rotation curves of spiral galaxies, and for small α can be written as

$$f(R) = \frac{(R + \Lambda)^{1+\alpha/2}}{R_c^{\alpha/2}}, \tag{17}$$

which reduces to $f(R) = R + \Lambda$ for $\alpha \ll 1$. In Solar system scale, in which the range of distances are sufficiently smaller than the length scale of the hybrid action, we obtain metric elements of Eq. (6) up to the first order terms in α as

$$B(r) = 1 - \frac{2GM}{c^2 r} + \frac{\alpha r}{d}$$
$$A(r) = \frac{1}{B(r)}, \tag{18}$$

therefor the weak field effective potential which comes from Eq. (15) obtains as

$$\phi(r) = -\frac{GM}{r} + \frac{\alpha c^2}{2d}r, \tag{19}$$

where the first term is the Newtonian potential and the second one is corrected term. Acceleration of a test particle in this potential is

$$a = -\frac{GM}{r^2} - \frac{\alpha c^2}{2d}, \tag{20}$$

where the second term in the right hand side of this equation, is a constant acceleration, independent of the central mass. We may correspond this extra term to the pioneer anomalous and constrain it with the observed value of $a_p = (8.47 \pm 1.33) \times 10^{-10} ms^{-2}$ which results in $\alpha/d \cong 10^{-}26m^{-1}$. In this case, correction to the gravitational potential is

$$\Delta\phi(r) = \frac{\alpha c^2}{2d}r, \tag{21}$$

where the acceleration of a test particle in this potential is

$$A_r = -\frac{\alpha c^2}{2d}. \tag{22}$$

On using

$$\int_0^{2\pi} (\cos E - e)dE = -2\pi e, \tag{23}$$

and Eqs. (9)-(12) it yields to the following perihelioin precession

$$< \dot{\varpi} >= -\frac{\alpha c^2}{2d}\frac{\sqrt{(1-e^2)a}}{\sqrt{GM}}. \tag{24}$$

It is important to note that $\dot{\varpi}$ depends on a positive power of the semi major axis a. The predicted extra precession of Eq. (24) can be fruitfully compared to the correction of usual Newtonian-Einsteinian perihelion rates of the planets of the Solar system phenomenological estimated by Pitjeva (2005a,b, 2006, 2008). According to the general outline of section 3, we will not use one perihelion at

a time for each planet. Indeed let us consider a pair of planets A and B, take the ratio of prediction of Eq. (24)

$$\frac{\dot{\varpi}_A}{\dot{\varpi}_B} = \sqrt{\frac{(1-e_A^2)M_B a_A}{(1-e_B^2)M_A a_B}}. \tag{25}$$

Now we consider a pair of planets A and B and take the ratio of their estimated extra-rates of perihelia in approximation small eccentricity; if Eq. (24) is responsible for them,

$$\Psi_{AB} = |\frac{\dot{\varpi}_A}{\dot{\varpi}_B} - (\frac{a_A}{a_B})^{\frac{1}{2}}|, \tag{26}$$

then the quantity must be compatible with zero. Table 3 shows the results of observational ratio and theoretical ratio of planets. It is important to note that the uncertainty in Ψ_{AB} has been conservatively estimated by linearity adding the individual terms coming from the propagation of the errors in $\dot{\varpi}$ and a in Eq. (26). In general, deviation of each function has the form of

$$\varrho_f = \sqrt{(\sum \partial f/\partial x_i)^2 (\varrho_i)^2}, \tag{27}$$

and errors in Ω_{AB} due to δa and $\delta\Delta\dot{\varpi}$ have the form below

$$\delta\Psi_{AB} \leq |\frac{\Delta\dot{\varpi}_A}{\Delta\dot{\varpi}_B}|(\frac{\delta\Delta\dot{\varpi}_A}{|\Delta\dot{\varpi}_A|} + \frac{\delta\Delta\dot{\varpi}_B}{|\Delta\dot{\varpi}_B|}) + \frac{1}{2}(\frac{a_A}{a_B})^{\frac{3}{2}}(\frac{\delta a_A}{a_A} + \frac{\delta a_B}{a_B}). \tag{28}$$

To see that how, this model acts in the Solar system, we can compare it with other corrections in Solar system. To this end we use power-law and logarithmic model in weak field approximation. Capozziello et al. (2007) started the Lagrangian of the form $f(R) = f_0 R^n$, in the metric approach, to obtain solutions to describe galaxies rotation curves without need for dark matter. They obtain modified gravitational potential of the form

$$\phi(r) = -\frac{GM}{r}[1 + (\frac{r}{r_c})^\beta], \tag{29}$$

where β relates to the the power of n in the modified Lagrangian via

$$\beta = \frac{12n^2 - 7n - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}. \tag{30}$$

This equation clearly leads to the radial acceleration

$$A_r = \frac{(\beta - 1)GM}{r_c^{\beta}} r^{\beta - 2} \tag{31}$$

It yields the following perihelion precession (Iorio, 2008)

$$< \dot{\varpi} >= \frac{(\beta - 1)\sqrt{M}}{2\pi r_c^{\beta}} a^{\beta - \frac{3}{2}}. \tag{32}$$

Capozziello et al. (2007) applied Eq. (29) to a sample of 15 low surfaces brightness galaxies with combined HI and $H\alpha$ measurements of the rotation curve extending in the putative dark matter dominated region. They obtained a good agreement between the theoretical rotation curves and the data using only stellar disk and interstellar gas when the slope n of the gravity Lagrangian is set to the value $n = 3.5$ (giving $\beta = 0.817$) obtained by fitting the SNeIa Hubble diagram with the assumed power law $f(R)$ model. For $\beta = 0.817$, results are in figure 1. Logarithmic correction suggested to solving the problem of dark matter in galaxies (Van Moorsel, 1987; Fabris & Campos, 2008; Sobouti, 2007). The potential is the form of

$$V_{ln} = -\alpha GM \ln(\frac{r}{r_0}), \tag{33}$$

that gives the radial extra acceleration

$$A_r = -\frac{\alpha GM}{r}, \tag{34}$$

and it gives the precession to the form of

$$< \dot{\varpi} >= -\alpha \sqrt{\frac{GM(1 - e^2)}{a}} (\frac{-1 + \sqrt{(1 - e^2)}}{e^2}). \tag{35}$$

The results for this model are in figure 1. As a conclusion the logarithmic and power law correction is ruled out by the present day observations (Iorio, 2008). As we see in figure 1, the correction to the form of this asymptotic behavior of hybrid $f(R)$, has the minimum difference with observational data.

Now the question is, about the properties of these models. we see that each of these corrections, has the term like r^n in it. For power law model $n = 0.817$, for logarithmic correction $n = -1$ and for the hybrid model $n = 0$. then we can debate on the power of r^n, and find that, which power of n has the appropriate accordance with observational data. With primary surveys we obtain $n = 5.38$.

For such potential, we can find the metric and the $f(R)$ model from inverse solution which is in preparation.

5 Deflection of light

To see that the effect of this asymptotic hybrid $f(R)$ model on the amount of deflection, we calculate the deflection of the orbit from a straight line

$$\Delta\Phi = 2|\Phi(r_0) - \Phi_{00}| - \pi,$$

where r_0 is the closet distance to the Sun.

$$\Delta\Phi = 2\int_0^\infty A^{1/2}(r)\left[\left(\frac{r}{r_0}\right)^2\left(\frac{B(r_0)}{B(r)} - 1\right)\right]^{-1/2}\frac{dr}{r} - \pi.$$

By using the values of $A(r)$ and $B(r)$, we could obtain $\Delta\Phi$ as elliptic integrals

$$\Delta\Phi = GR_{terms} - \frac{1}{2}\frac{\alpha c^2}{d}r_0[\ln\frac{r}{r_0} + \ln\sqrt{(\frac{r}{r_0})^2 - 1}]$$

$$+ \frac{1}{2}\frac{\alpha c^2}{d}[r_0\ln\frac{r}{r_0} + \ln\sqrt{(\frac{r}{r_0})^2 - 1}] - \frac{r_0^2}{r + r_0}\sqrt{(\frac{r}{r_0})^2 - 1}]. \tag{36}$$

We replace r with r_0, then to first order in MG/r_0 the deflection is

$$\Delta\Phi = \frac{4MG}{r_0} \tag{37}$$

and this is the prediction of GR. Then this asymptotic behavior of hybrid $f(R)$ model do not violate GR regimes and have no any additional effect on deflection of light.

6 Stability of circular orbits

Finally in the context of stability condition of orbits, we test the obtained weak field potential with numerous equation of stability:

$$\frac{F_c'(r)}{F_c(r)} + \frac{3}{r} > 0, \tag{38}$$

10

in which F_c is central force, we obtain $r^2 < GMd/3\alpha c^2$. Then maximum of the radius of stable circular orbit, is about $r_s \approx 2.22 \times 10^{11}$ km which is 10^3 larger than typical Solar system radius, in order of magnitude. Therefore all the planets in this modified regime may have stable orbits.

7 Comments and conclusions

In this paper we have studied the secular precession of the pericenture of a test particle in motion around a central mass M whose Newtonian gravitational potential exhibits a correction which is to form of Eq. (19). In order to put on the test the hypothesis that such extra forces are not zero we devised a suitable test by taking into account the ratios of the corrections to the secular precession of perihelia estimated by Pitjeva (2005a,b, 2006, 2008). The results obtained, resumed by Eq. (24), show that modifications of the Newtonian potentials like this examined in this paper are compatible with the currently available apsidal extra-precession of the Solar system planets. Of course we have to say that this model will be ruled out if we take the inverse ratio of precession for planets. Hence we take this regime that, take the inner planet to outer planet. Our correction can be used to describe the problem of dark matter in the Solar system, if we choose the appropriate values for parameters of the model. It gives Pioneer anomalous acceleration for the suitable value of distance. When we use the other estimative corrections to Newtonian-Einstenian planetary perihelion rates, maybe obtain better compatibility for this model in the Solar system. We had shown that this asymptotic hybrid model give the GR, deflection of light and had no extra effect on it. From comparison alternative models, we could offer the generic model of potential, and obtain it's action. This proposal model has minimum variance, and our aim is checking its compatibility with dark matter and dark energy data set.

Acknowledgments: We would like to thank anonymous referee for useful comments.

References

Anderson, J.D., et al., Indication, from Pioneer 10/11, Galileo, and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration, Phys. Rev. Lett. 81, 2858, 1998

Anderson, J.D., et al., Study of the anomalous acceleration of Pioneer 10 and 11, Phys. Rev. D 65, 082004, 2002

Bertolami, O., Paramos, J., & Turyshev, S.G., General Theory of Relativity: Will it survive the next decade? "Lasers, Clocks, and Drag-Free: Technolo-

gies for Future Exploration in Space and Tests of Gravity", H. Dittus, C. Laemmerzahl, S. Turyshev, eds. pp. 27-67 (Springer Verlag, 2006)

Bertolami, O., Int. J. Mod. Phys. D 16, 2003, 2008

Capozziello, S., Cardone, V.F., & Troisi, A., Low surface brightness galaxies rotation curves in the low energy limit of R^n gravity: no need for dark matter?, Mon. Not. Roy. Astron. Soc. 375, 1423, 2007

Capozziello, S., & Francaviglia, M., Extended Theories of Gravity and their Cosmological and Astrophysical Applications. Gen. Rel. Grav. 40, 357, 2008

Clowe, D., et al., A Direct Empirical Proof of the Existence of Dark Metter, Astrophys. J. 648, L 109, 2006

Cole, S., et al., The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final dataset and cosmological implications, Mon. Not. Roy. Astron. Soc. 362, 505, 2005

de la Cruz-Dombriz, A., & Dobado, A., $f(R)$ gravity without a cosmological constant, Phys. Rev. D 74, 087501, 2006

Fabris, J.C., & Campos, J.P., Spiral galaxies rotation curves with a logarithmic corrected newtonian gravitational potential, Gen. Relativ. Gravit., 10.1007/s10714-008-0654-0, 2008

Faraoni, V., $f(R)$ Theories Of Gravity, Rev. Mod. Phys. 82, 451, 2010

Frigerio, C., & Salucci, P., Analysis of rotation curves in the framework R^n gravity, Mon. Not. Roy. Astron. Soc. 381, 1103, 2007

Iorio, L., Solar System tests of some models of modified gravity proposed to explain galactic rotation curves without dark matter Matteo Luca Ruggiero, Scholarly Research Exchange, vol . 2008, article ID 968393,

Lammerzahl, C., Preuss, O., & Dittus, H., Is the physics within Solar system really understood?, in H.Dittus, C. Lammerzahl and S. G, Turyshev (ed.) Lasers, Clocks and Drag Free control: Exploration of Relativistic Gravity in Space, Springer, Berlin, 2008, pp. 75-104.

Magnano, G., Are there metric theories of gravity other than general relativity?, Talk given at the XI Italian Conference on General Relativity and Gravitation, Trieste, Sept 26-30, 1994

Multamaki, T., & Vilja, I., Spherically symmetric solutions of modified field equation in $f(R)$ theories of gravity , Phys. Rev. D 74, 064022, 2006

Nieto, M.M., New Horizons and the Onset of the Pioneer Anomaly, Phys. Lett. B 659, 483, 2008

Nojiri, S., & Odintsov, S.D., Introduction to Modified Gravity and Gravitational Alternative for Dark Energy, Int. J. Geom. Meth. Mod. Phys. 4, 115, 2007

Nojiri, S., & Odintsov, S.D., Modified gravity as realistic candidate for dark energy, inflation and dark matter. AIP Conf. Proc. 1115, 212, 2009

Perlmutter, S., et al.,Measurements of Ω and Λ from 42 High-Redshift Supernovae, Astrophys. J 517, 565, 1999

Pitjeva, E.V., High-Precission Ephemerides of Planets-EPM and Determination of some Astronomical Constants, Sol. Syst. Res. 39, 176, 2005

Pitjeva, E.V., Relativistic Effects and Oblateness from Radar Observations of

Planets and Spacecraft, Astron. Lett. 31, 340, 2005

Pitjeva, E.V., The dynamical model of the planet motions and EPM ephemerides, in Nomenclature, Precession and New Models in Fundamental Astronomy, 26th Meeting of the IAU, Joint Discussion 16, Prague, Czech Republic, August 2006

Pitjeva, E.V., Use of optical and radio astrometric observations of planets, satellites and spacecraft for ephemeris astronomy Proceedings of the International Astronomical Union, Volume 3, Symposium S248, October 2007, pp 20-22 doi:10.1017/S1743921308018565 (About doi), Available on CJO 08 Jul 2008

Riess, A.G., et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant, The Astronomical Journal, vol. 116, no. 3, pp. 1009, 1998.

Roy, A.E., Orbital motion, IOP Publishing, Bristol, 2005

Saffari, R., & Rahvar, S., $f(R)$ Gravity: From the Pioneer Anomaly to the Cosmic Acceleration, Phys. Rev. D 77, 104028, 2008

Sobouti, Y., An $f(R)$ gravitation for galactic environments, Astron. Astrophys. 464, 921, 2007

Sotiriou, T., & Faraoni, V., f(R) Theories of Gravity, Rev. Mod. Phys. 82, 451, 2010

Spergel, D.N., et al., First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters, Astrophys. J. Supple. 148, 175, 2003

Spergel, D.N., et al., Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology, ApJS 170, 377, 2007

Tegmark, M., et al., Cosmological constraints from the SDSS luminous red galaxies, Phys. Rev. D 74, 123507, 2006

Van Moorsel, G.A., Dark matter associated with binary galaxies, A.&A., vol. 176, no. 1, pp. 1324, 1987.

Table 1
observational data for inner planets

planet	Mercury	Earth	Mars
$\Delta\dot{\varpi}(arcs/cy)$	-0.000036±.005	-0.0002±.00004	0.0001±.0005
$a(AU)$	0.387	1	1.523
e	0.205	0.016	0.0934
$P(yr)$	0.24	1	1.88

Table 2
observational data for outer planets

planet	Jupiter	Saturn	Uranus
$\Delta\dot{\varpi}(arcs/cy)$	0.0062±.036	-0.92±2.9	0.57±13
$a(AU)$	5.203	9.537	19.191
e	0.0483	0.0541	0.0471
$P(yr)$	11.86	29.45	84.07

Table 3
results, A B denotes the pair of planets used $\Omega_{AB} = \frac{\Delta\dot{\varpi}_A}{\Delta\dot{\varpi}_B}$, $\xi_{AB} = (\frac{a_A}{a_B})^{\frac{1}{2}}$. The perihelion extra rates for the planets have been retrieved from Pitjeva (2006), their errors are not formal; and re scaled by a factor 10. The uncertainties in the semi major axis have been retrieved from table 5 of Pitjeva (2005a)

| Pairs	A	B	Ω_{AB}	ξ_{AB}	Ψ_{AB}
1	Mer	Jup	-0.6±4.1	0.272± 10^{-9}	0.87±4.1
2	Ear	Jup	-0.03±0.25	0.438± 10^{-8}	0.46±0.25
3	Mar	Jup	0.02±0.17	0.541± 10^{-9}	0.52±0.17
4	Mer	Sat	0.004±0.017	0.201± 10^{-8}	0.19±0.017
5	Ear	Sat	0.0002±0.0011	0.323± 10^{-10}	0.32±0.0011
6	Mar	Sat	-0.0001±0.0009	0.399± 10^{-8}	0.399±0.0009
7	Jup	Sat	-0.006±0.060	0.738± 10^{-8}	0.73±0.060
8	Mer	Ura	-0.006±0.152	0.142± 10^{-8}	0.146±0.152
9	Ear	Ura	0.0003±0.0087	0.228± 10^{-9}	0.21±0.0087
10	Mar	Ura	0.0002±0.0048	0.281± 10^{-8}	0.27±0.0048
11	Jup	Ura	0.01±0.31	0.52± 10^{-9}	0.51±0.31
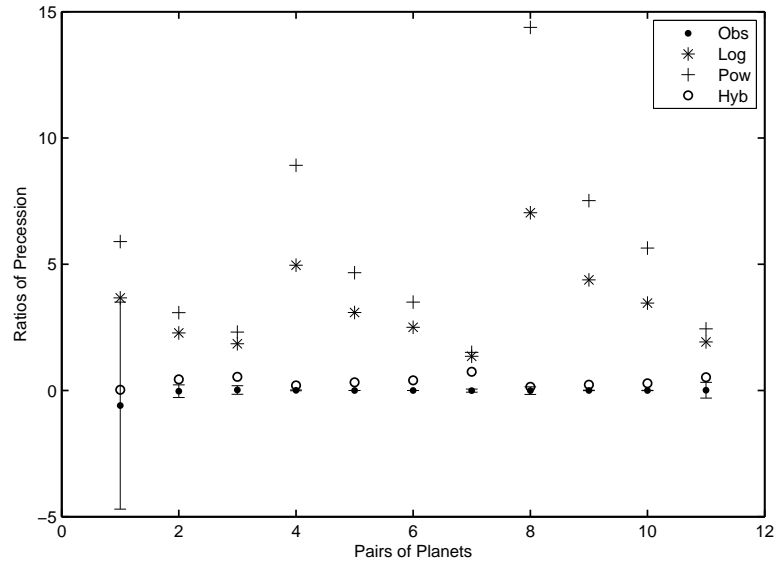
Fig. 1. This figure shows accordance of observational precession with hybrid action theoretical results against power-low and logarithmic corrections.